Received: *March 17, 2001,* Accepted: *April 1, 2001*

Unstable solitons in noncommutative gauge theory

Mina Aganagic, Rajesh Gopakumar, Shiraz Minwalla and Andrew Strominger

Jefferson Physical Laboratory, Harvard University
Cambridge, MA 02138, USA
E-mail: mina@cartan.harvard.edu, gopakumr@pauli.harvard.edu,
minwalla@schwinger.harvard.edu andy@pauli.harvard.edu

Abstract: We find a class of exact solutions of noncommutative gauge theories corresponding to unstable non-BPS solitons. In the two-dimensional euclidean (or 2+1 dimensional lorentzian) U(1) theory we find localized solutions carrying nonzero magnetic flux. In four euclidean dimensions we find non-BPS solutions with the same Pontrjagin charge but greater energy than the usual self-dual Yang-Mills instanton. We conjecture that these solutions and generalizations thereof correspond to non-supersymmetric configurations of D$(p-2k)$-branes (or intersections thereof) in a Dp-brane in the noncommutative scaling limit of large B-field. In the particular case of a 0-brane on a 2-brane the analysis of small fluctuations reveals an infinite tower of states which agrees exactly with that of the 0-2 CFT in the scaling limit. The spectrum contains a tachyon, and we show explicitly that the endpoint of tachyon condensation corresponds to the 0-brane dissolved in the 2-brane.

Keywords: Field Theories in Lower Dimensions, Solitons Monopoles and Instantons, D-branes, Non-Commutative Geometry.

JHEP04(2001)001

Contents

1. Introduction

The study of classical solutions of noncommutative field theories has led to a number of surprises and new insights. Instanton solutions to four-dimensional noncommutative Yang-Mills theory, that elegantly resolve singularities in the moduli space, were found in [1]. Solitons in a wide class of scalar field theories were found in [2]. These carry no topological charge and do not saturate a BPS bound. Other interesting solutions have been discussed in [3]–[12].

In this paper we construct solitonic solutions of non-commutative gauge theories. We find exact classical solutions which carry various kinds of fluxes but, as in [2], are not, in general, BPS (in contrast to the gauge theory solutions in [1]). In the context of string theory we argue that they correspond to non-supersymmetric configurations of branes localized on other branes. In general it is difficult to find exact non-BPS solutions in an interacting gauge theory. Here it is possible using the representation of noncommutative gauge fields as operators.

The first solution we construct is a soliton in $2+1$ dimensional noncommutative U(1) gauge theory. It carries a unit of magnetic flux and has finite localized energy. No such soliton exists in an ordinary (commutative) gauge theory. We conjecture that the noncommutative solution corresponds to an undissolved 0-brane on a 2-brane. Evidence for this conjecture is obtained from a comparison of energies as well as the spectrum of fluctuations with the corresponding CFT. We find a match in full detail with the 0-2 system in the appropriate noncommutative Yang Mills scaling limit. The spectrum of the 0-2 strings consists, in this limit, of a single oscillator tower of massive states with the expected tachyon at the bottom. We further work out the complete lagrangian for the modes relevant to condensing the tachyon which allows us to follow this process to its endpoint.[1] This is seen to correspond to the expected configuration of a 0-brane dissolved in the 2-brane.

We also construct new solutions in 4-dimensional euclidean noncommutative gauge theory with a single unit of Pontrjagin charge. The solutions have the opposite Pontrjagin charge of the solutions of [1] . They are non BPS at generic Θ but BPS when Θ is selfdual. We will argue that this fits perfectly with their interpretation as 0-branes on a 4-brane.

A variety of other solutions, corresponding to multiple, intersecting, and higher codimension branes within branes are briefly described.

It may be useful to contrast the present work with the construction of D-branes as noncommutative tachyon solitons [14, 15, 5, 12]. Following [16], these papers construct D-branes as tachyon lumps (in the bosonic theory) or tachyon vortices (in the supersymmetric theory) asymptoting to the closed string vacuum. In this paper we construct branes as flux lumps within higher-dimension branes (following [17, 18]). In describing branes as tachyon lumps or vortices, one faces the twin difficulties of the lack of a controlled approximation justifying the dynamics and of understanding the enigmatic nothing state outside the brane. Both these difficulties are circumvented in this paper. The effective dynamics we use is systematically derived in a limit [19]

[1] An analysis of tachyon condensation in string theory, complementary to the one here in the non-commutative gauge theory, was carried out in [13] in essentially the same system.

in which the string scale is sent to infinity while keeping the noncommutativity scale fixed. Further, our solitons asymptote to the well understood vacuum on higher dimensional branes, rather than the mysterious nothing state, which, unfortunately, cannot be studied in our setting. Nevertheless the system we consider is quite rich and does contain tachyons and tachyon condensation.

The reader who is interested only in the classical solutions and not their interpretation in string theory, need consult only sections 2 and 3. Technical details are presented in in several appendices. As this work was nearing completion, an interesting paper [20] appeared which also presents the solution of section 2.1.

Note added: After the first version of this paper appeared we learnt that the solutions in section 2.1 had been first written down by Polychronakos [21].

2. Classical solutions in $2+1$ dimensional gauge theory

In this section we present a set of exact localized solutions to the equations of motion of spatially noncommutative Yang Mills theory in 2+1 dimensions. In subsection 2.1 we present solutions for localized lumps of flux in pure Yang Mills theory. In subsection 2.2 we explain how the soliton moduli space is enlarged in theories with additional scalar fields in the adjoint (e.g. Super Yang Mills with 16 supercharges). In subsection 2.3 we analyze the spectrum of small fluctuations about these solitonic lumps.

2.1 Codimension two flux lumps in pure Yang Mills

Consider U(1) Yang Mills on a $2+1$ dimensional noncommutative space with spatial noncommutativity

$$[z, \bar{z}] = i\Theta^{z\bar{z}} = \theta\,. \tag{2.1}$$

We will take the metric $G_{\mu\nu}$ to be diagonal, and let $G_{00} = -1$, $G_{z\bar{z}} = G$ (we will mostly set $G = 1$ reinstating it when necessary using coordinate invariance).[2] In the temporal $A_0 = 0$ gauge, the Yang Mills action is

$$\begin{aligned} -S &= \frac{1}{4g_{YM}^2}\int d^3x\,(F_{\mu\nu}F^{\mu\nu}) \\ &= \frac{2\pi\theta}{g_{YM}^2}\int dt\,\mathrm{Tr}\left[-\partial_t\bar{C}\partial_t C + \left([C,\bar{C}] + \frac{1}{\theta}\right)^2\right], \end{aligned} \tag{2.2}$$

where we have set $C_z = C = -iA_z + a^\dagger$, $C_{\bar{z}} = \bar{C}$, $[a, a^\dagger] = 1/\theta$.

The equations of motion for the dynamical field C_z in the $A_0 = 0$ gauge is

$$\partial_t^2 C = [C, [C, \bar{C}]]\,. \tag{2.3}$$

[2]We use the notations and conventions of [12] (see especially section 2.2) in what follows.

In this gauge one must also impose the Gauss law constraint

$$[\bar{C}, \partial_t C] + [C, \partial_t \bar{C}] = 0 \tag{2.4}$$

obtained by varying (2.2) with respect to A_0 prior to gauge fixing, and then setting $A_0 = 0$.

We will find a series of static localized solutions to (2.3), labeled by a positive integer corresponding to the number of units of flux they carry. The solution with $m > 0$ units of flux is

$$C = C_0 \equiv (S^\dagger)^m a^\dagger S^m + \sum_{a=0}^{m-1} c^a |a\rangle\langle a| \,, \tag{2.5}$$

where c^a are arbitrary real numbers and the shift operator $S = \sum_{i=0}^{\infty} |i\rangle\langle i+1|$. S obeys

$$\begin{aligned} SS^\dagger &= 1\,, & S^\dagger S &= 1 - P_0\,, \\ S^m (S^\dagger)^m &= 1\,, & (S^\dagger)^m S^m &= 1 - P_{m-1}\,, \end{aligned} \tag{2.6}$$

where $P_{m-1} = \sum_{i=0}^{m-1} |i\rangle\langle i|$ is the rank m projector. It is useful to note that[3] $S^m|k\rangle = |k-m\rangle$, $(S^\dagger)^m|k\rangle = |k+m\rangle$. The matrix C_0 takes the block form

$$C_0 = \begin{pmatrix} c_0 & 0 \\ 0 & a^\dagger \end{pmatrix} . \tag{2.7}$$

Here the upper left hand $m \times m$ block c is a diagonal matrix $(c_0)_{ab} = c^a \delta_{ab}$.

The flux operator $-iF_{z\bar{z}} = F_0$ evaluated on C_0 is given by

$$\theta F_0 = 1 + \theta[C_0, \bar{C}_0] = 1 + \theta (S^\dagger)^m [a^\dagger, a] S^m = P_{m-1}\,. \tag{2.8}$$

For an arbitrary configuration, the normalized integral of the flux over the z plane may be rewritten as a trace over the operator F

$$c_1 = \frac{1}{2\pi} \int F = \theta \,\mathrm{Tr}\, F\,; \tag{2.9}$$

from (2.8) and (2.9) C_0 carries m units of flux. Since $S^m|a\rangle = 0 = \langle a|(S^\dagger)^m$ $(a = 0, \ldots, m-1)$

$$[C_0, F_0] = 0\,, \tag{2.10}$$

and C_0 is a static solution to the equation of motion (2.3). Its energy is

$$E = \frac{2\pi\theta}{2g_{YM}^2} \,\mathrm{Tr}\, F_0^2 = \frac{m\pi}{g_{YM}^2 \theta}\,. \tag{2.11}$$

Note that the generalization of (2.2) to a constant background metric (of the form described at the beginning of section 2.1) has solitons of energy

$$E = \frac{m\pi}{g_{YM}^2 \sqrt{\Theta^2}}\,, \qquad \Theta^2 = \Theta^{\mu\nu}\Theta^{\alpha\beta} G_{\mu\alpha} G_{\nu\beta}\,. \tag{2.12}$$

In section 4 we will interpret (2.5) as the solution corresponding to m D0-branes at positions c^i on a D2-brane.

[3] Here and through the rest of this paper, we will use the convention $|i\rangle = 0$ when i is negative.

2.2 Generalization with adjoint scalar fields

We will now generalize the solutions of the previous subsection to solutions of $2+1$ Yang Mills interacting with t transverse scalars φ^i. Picking the temporal gauge

$$
\begin{aligned}
-S &= \frac{1}{4g_{YM}^2}\int d^3x \left(F_{\mu\nu}F^{\mu\nu} + 2\sum_{i=1}^{t} D_\mu\varphi^i D^\mu\varphi^i + \sum_{i,j=1}^{t}[\varphi^i,\varphi^j][\varphi^i,\varphi^j]\right) \\
&= \frac{2\pi\theta}{g_{YM}^2}\int dt\,\mathrm{Tr}\left[-\partial_t\bar{C}\partial_t C - \sum_{i=1}^{t}\frac{1}{2}\partial_t\varphi^i\partial_t\varphi^i + \frac{1}{2}\left([C,\bar{C}]+\frac{1}{\theta}\right)^2 + \right. \\
&\qquad \left. + \sum_{i=1}^{t}[C,\varphi^i][\varphi^i,\bar{C}] + \frac{1}{4}\sum_{i,j=1}^{t}[\varphi^i,\varphi^j][\varphi^j,\varphi^i]\right]. \qquad (2.13)
\end{aligned}
$$

The counterpart of the equation of motion (2.3) is

$$
\begin{aligned}
\partial_t^2 C &= [C,[C,\bar{C}]] + \sum_{i=1}^{t}[\varphi^i,[C,\varphi^i]] \\
\partial_t^2\varphi^i &= [C,[\varphi^i,\bar{C}]] + [\bar{C},[\varphi^i,C]] + \sum_{j=1}^{t}[\varphi^j,[\varphi^i,\varphi^j]]\,, \qquad (2.14)
\end{aligned}
$$

while the Gauss law constraint (2.4) becomes

$$
[\bar{C},\partial_t C] + [C,\partial_t\bar{C}] + \sum_{i=1}^{t}[\varphi^i,\partial_t\varphi^i] = 0\,. \qquad (2.15)
$$

The solutions (2.5) of the pure Yang Mills theory generalize to

$$
\begin{aligned}
C = C_0 &= (S^\dagger)^m a^\dagger S^m + \sum_{a=0}^{m-1} c^a|a\rangle\langle a| \\
\varphi_0^i &= \sum_{a=0}^{m-1}\varphi_a^i|a\rangle\langle a|\,, \qquad (2.16)
\end{aligned}
$$

where c^a and φ_a^i are arbitrary real numbers. Note that the gauge field in (2.16) is identical to that in (2.5); in particular the solutions (2.16) also carry m units of flux. (2.16) takes the form

$$
C_0 = \begin{pmatrix} c_0 & 0 \\ 0 & a^\dagger \end{pmatrix}, \qquad \varphi_0^i = \begin{pmatrix} \varphi_0^i & 0 \\ 0 & a^\dagger \end{pmatrix}, \qquad (2.17)
$$

where c_0 and φ_0^i are $m\times m$ diagonal matrices, $(c_0)_{ab} = c^a\delta_{ab}$ and $(\varphi_0^i)_{ab} = \varphi_a^i\delta_{ab}$. As

$$
[C_0,F] = 0\,, \qquad [C_0,\varphi_0^i] = 0\,, \qquad [\varphi_0^i,\varphi_0^j] = 0\,, \qquad (2.18)
$$

(2.16) solves the equations of motion (2.3) and its energy is independent of φ_0 and is given by (2.12). ϕ_a^i will be interpreted as the transverse positions of the m 0-branes in section 4.

JHEP04(2001)001

2.3 Small fluctuations

In appendix A we have computed the spectrum of quadratic fluctuations about the solutions (2.16). In this subsection we list the results of that computation. We first describe the spectrum about the special solution $\varphi_0^i = 0$, $c_0^a = 0$ and then explain how the spectrum is modified on giving φ^i a vacuum expectation value.

Let

$$\begin{aligned} C &= C_0 + \delta C \\ \varphi^i &= \delta\varphi^i \end{aligned} \tag{2.19}$$

and let

$$\delta C = \begin{pmatrix} A & W \\ \bar{T} & D \end{pmatrix}, \qquad \delta\varphi^i = \begin{pmatrix} \chi^i & \psi^i \\ \bar{\psi}^i & \gamma^i \end{pmatrix}. \tag{2.20}$$

In appendix A we expand the potential energy in (2.13) to quadratic order in the fields $A, W, T, D, \chi^i, \psi^i, \gamma^i$, and diagonalize this quadratic form. We find that:

a. The fields A and χ^i do not appear in the quadratic potential and so are massless modes in 0+1 dimensions. In section 4 these modes will be identified with the scalar fields living on the world volume of m coincident D0-branes.

b. The potential for the modes D and γ^i is precisely that for fluctuations of the vector and the transverse scalars of (2.2) about its vacuum. In section 4 D and γ^i will be identified with the modes on a D2-brane.

c. The off diagonal gauge field fluctuation modes W and T mix with each other. These fields carry $2m \times \infty$ complex degrees of freedom in 0+1 dimensions. Half of these modes are pure gauge, and are set to zero by the Gauss Law constraint. The other half form a tower of states. Each energy level of the tower has m complex fields in the fundamental of $\mathrm{U}(m)$. The base of the tower is tachyonic with the m complex modes $\langle a|T|m\rangle$ $(a = 0, \ldots, m-1)$ of mass $-1/\theta$. The rest of the tower has a harmonic oscillator spectrum, with energies

$$m_k^2 = \frac{(2k+1)}{\theta}, \qquad k = 1, 2, \ldots \tag{2.21}$$

The modes of the hermitian field ψ^i also arrange themselves into a harmonic oscillator tower of complex fields in the fundamental of $\mathrm{U}(m)$. The tower consists of states of energy

$$m_k^2 = \frac{(2k+1)}{\theta}, \qquad k = 0, 1, \ldots \tag{2.22}$$

The tower of states found here is similar to that found in [22]. All these modes will be identified with the modes of 0-2 strings in section 4. For later use we note that, in the more general backgound metric described at the beginning of section 2.1, the fluctuation spectrum is obtained by replacing θ in (2.21), (2.22) with $\sqrt{\Theta^2}$ where Θ^2 is defined in (2.12).

On turning on φ_0^1 vev (for simplicity keeping c_0 and $\varphi_0^i = 0$ for $i = 2 \ldots t$), the spectrum described above is modified (see appendix A for details):

1. the Gauss law constraint sets off diagonal elements in χ^1 to zero; diagonal components of χ^1 continue to be massless physical excitations. The fields A_{ab} and χ^i_{ab} $(i = 2, \ldots, t)$ $(a \neq b)$ become massive, with squared mass

$$m^2(A_{ab}) = m^2(\chi^i_{ab}) = (\varphi_a - \varphi_b)^2 \,. \tag{2.23}$$

   Thus a φ_a vev activates the Higgs mechanism; the off diagonal components of χ^1 are eaten by a 'vector' field which becomes massive.

2. the 2-2 sector is unaffected by φ^1.

3. the φ_1 vev has a simple universal effect on the spectrum of fluctuations of all off diagonal modes ((0-2) strings). The squared mass of every mode with index $a(a = 1, \ldots, m-1)$ is increased by $(\varphi_a)^2$.

3. More solutions in higher dimensions

3.1 Codimension four solutions

In this subsection we describe solutions of four-dimensional euclidean noncommutative gauge theories which have the same Pontrjagin charge as the usual Yang-Mills instanton, but in general higher action (or energy). To start with we consider the U(1) gauge theory with no additional matter fields.

In four dimensions the two complex shifted gauge fields C_m, with $m = 1, 2$ are naturally viewed as operators on the quantum mechanical Hilbert space of two particles with creation and annihilation operators obeying

$$[a_m^\dagger, a_{\bar{n}}] = -i\Theta^{-1}_{m\bar{n}} \,. \tag{3.1}$$

The noncommutative field strength is

$$F_{m\bar{n}} = i[C_m, C_{\bar{n}}] - \Theta^{-1}_{m\bar{n}} \,. \tag{3.2}$$

The gauge field equation of motion can be written

$$[C^m, [C_m, C_{\bar{n}}]] = 0 \,. \tag{3.3}$$

A solution with a single unit of Pontrjagin charge is provided by

$$C_m = T^\dagger a_m^\dagger T\,, \tag{3.4}$$

where T obeys

$$TT^\dagger = 1\,, \qquad T^\dagger T = 1 - P_0\,, \qquad P_0 T^\dagger = T P_0 = 0\,. \tag{3.5}$$

In this expression P_0 is a projection on to the lowest radial wave functions in *both* complex directions, i.e. a four-dimensional Gaussian centered at the origin. There are a number of gauge equivalent expressions for T, we give an example in appendix B. The field strength for this configuration is easily seen to be

$$F_{m\bar{n}} = -\Theta^{-1}_{m\bar{n}} P_0\,, \tag{3.6}$$

which obeys the equation of motion as a result of (3.5). Note that F is (anti) self-dual if and only if Θ is (anti) self-dual. The Pontrjagin charge and action for the solution (3.4) are

$$p_1 = \frac{1}{8\pi^2}\int d^4x F\wedge F = \frac{1}{2}|\sqrt{\det\Theta}|\Theta^{-1}\wedge\Theta^{-1} = \pm 1 \tag{3.7}$$

$$\frac{g_{YM}^2}{4\pi^2}S = \frac{1}{8\pi^2}\int d^4x F^{m\bar{n}}F_{m\bar{n}} = \frac{1}{2}|\sqrt{\det\Theta}|\Theta^{-1}_{m\bar{n}}(\Theta^{-1})^{m\bar{n}}\,. \tag{3.8}$$

The RHS of (3.8) is equal to the Pontrjagin charge (and so the BPS bound is saturated) if and only if Θ is self dual or anti-self-dual.

A solution of noncommutative $\mathrm{U}(N)$ Yang Mills with a single unit of Pontrjagin charge is easily constructed by embedding (3.4) into a $\mathrm{U}(1)$ subgroup of $\mathrm{U}(N)$.

Let us first take the noncommutativity to be self-dual ($\Theta^- = 0$) so that the solution (3.4) is BPS, with a self-dual field strength $F^- = 0$. This differs from the solution of Nekrasov and Schwarz which is an anti-self-dual instanton with selfdual noncommutativity.[4] Recall that the self-dual instanton moduli space is a function only of Θ^- [19] and so, in particular, is the usual instanton moduli space when $\Theta^- = 0$. The embedding of (3.4) into $\mathrm{U}(N)$ is a self-dual instanton on the singular locus[5] in the self-dual instanton moduli space, usually referred to as the zero size instanton singularity. It is interesting that while the self-dual instanton moduli space is singular, our solution is smooth in the presence of noncommutativity.

For generic Θ, the solution (3.4) is neither self-dual nor anti-self-dual and does not saturate the BPS bound. If Θ takes the block diagonal form $\Theta^{\mu\nu} = (\theta_1\epsilon, \theta_2\epsilon)$,

[4] To the best of our knowledge, explicit instanton or anti-instanton solutions for generic Θ, or even generic self-dual instanton solutions for $\Theta^- = 0$, have not been found.

[5] In the $N = 4$ SYM theory (3.4) admits BPS deformations corresponding to turning on the transverse scalars $\varphi^i = \zeta^i|0\rangle\langle 0|$; these deformations correspond to moving along the Coulomb branch (taking the anti 0-brane away from the 4-brane: recall that the anti 0-brane, 4-brane system is SUSY when $\Theta^- = 0$).

JHEP04(2001)001

(where ϵ is the 2×2 antisymmetric matrix with $\epsilon^{12} = 1$), the action of (3.4) exceeds that of the corresponding BPS solution by

$$S - S_{BPS} = S - \frac{4\pi^2}{g_{YM}^2} = \frac{2\pi^2}{g_{YM}^2}\left(\sqrt{\frac{\theta_2}{\theta_1}} - \sqrt{\frac{\theta_1}{\theta_2}}\right)^2 . \tag{3.9}$$

It is surprising that exact, non-BPS solutions of the gauge theory can be analytically constructed. We conjecture that, in string theory in the appropriate scaling limit, (3.4) corresponds to an anti 0-brane sitting on top of a 4-brane. The fact that (3.4) is not supersymmetric is a consequence of the fact that the anti 0-brane 4-brane system is not supersymmetric at generic Θ. We will present some evidence for this conjecture in subsection 4.6.

We expect the spectrum of small fluctuations about (3.4) at generic Θ to include a tachyon that becomes massless at selfdual noncommutativity. At selfdual noncommutativity, (in the U(N) theory, for $N > 1$), the 'tachyon hypermultiplet' potential is expected to have flat directions, corresponding to the moduli space of dissolved self-dual instantons. It thus seems likely that the smooth solution (3.4) will permit smooth deformations into either the Higgs or the Couloumb branch of moduli space. It would be interesting to investigate this in more detail.

3.2 Generalizations

These solutions can be generalized in several ways. First, we can translate the instanton to the position r_m by adding the term $r_m P_0$ to C_m in (3.4). Raising the T and $T^\dagger$ on the right hand side of (3.4) to the mth power gives a solution with Pontrjagin charge m corresponding to m solitons at the origin. The $a^\dagger$ on the right hand side of (3.4) can be replaced by any solution of the equations of motion (3.3) — for example the general instanton solutions of [1]. Finally the construction easily generalizes to higher dimensions. One simply finds a T obeying (3.5) where P_0 is a higher dimensional projection operator. Both higher codimension and intersecting brane configurations may be described in this manner.

4. D-brane Interpretation

In this section we present evidence for the conjecture that the solitonic solutions constructed in previous sections represent branes on branes.

4.1 The 0-2 system

Consider a D2-brane extended in the 012 directions. Let the closed string metric and the B-field in the 12 directions be $g_{\mu\nu} = g\delta_{\mu\nu}$ and $B_{\mu\nu} = B\epsilon_{\mu\nu}$. The low energy effective action on such a brane is [23] noncommutative Yang Mills with 16 supercharges, Yang Mills coupling g_{YM}^2, open string metric $G_{\mu\nu} = G\delta_{\mu\nu}$ (we will not set $G = 1$ in this section) and noncommutativity parameter $\Theta^{\mu\nu} = \theta\epsilon^{\mu\nu}$ in the 12 directions where [19]

$$\begin{aligned}\theta &= \frac{1}{B}\\ G &= \frac{(2\pi\alpha' B)^2}{g}\\ g_{YM}^2 &= \frac{g_{str} 2\pi\alpha'^{1/2} B}{g}\,.\end{aligned} \tag{4.1}$$

In (4.1) we have retained only the leading terms in the limit of that is of interest to us in this paper, namely [19]

$$\alpha' B \gg g\,. \tag{4.2}$$

(4.2) is the limit in which the noncommutativity length scale is much larger than the string scale

$$\Theta^2 = \Theta^{\mu\nu}\Theta^{\alpha\beta} G_{\mu\alpha} G_{\nu\beta} = \frac{32\pi^4 \alpha'^4 B^2}{g^2} \gg \alpha'^2\,. \tag{4.3}$$

In this limit, all stringy corrections are irrelevant to processes that occur at the length and energy scale of noncommutativity, and such processes are accurately described in the low energy effective theory, namely noncommutative Yang Mills.

More specifically, in the spirit of [16]–[18], a zero brane sitting on top of a 2-brane may be expected to be represented as a soliton in the 2-brane string field theory. It will turn out that the size of this soliton is set by the noncommutativity scale,[6] and so is much larger than the string scale in the limit (4.2). Further, as this soliton carries a single unit of flux the field strength associated with this soliton is small in string units $\alpha'^2 G^{\mu\nu} G^{\alpha\beta} F_{\mu\alpha} F_{\nu\beta} \sim g^2/(\alpha'^2 B^2) \ll 1$. Thus this 0-2 system and its low energy dynamics, may be studied in detail using noncommutative Yang Mills.

We conjecture that the solution (2.5) in the world volume theory of the D2-brane represents m D0-branes at transverse positions $x_a^i = 2\pi\alpha'\varphi_a^i$.[7] We will now present evidence for this conjecture.

4.2 Matching energies

In this subsection we compare the energy of the solution (2.5) with the energy of a 0-brane sitting on top of the 2-brane. We find exact agreement.

The energy (2.12) of (2.5), as a function of closed string parameters is (for $m = 1$)

$$E = \frac{1}{2 g_{str}\sqrt{\alpha'}} \frac{g^2}{(2\pi\alpha' B)^2}\,. \tag{4.4}$$

[6] This is not true of every soliton in this theory. The anti D0-brane is a string scale soliton, and so cannot be described in the low energy effective field theory.

[7] This interpretation for the scalar field part of (2.5) was advanced by J. Maldacena (unpublished).

JHEP04(2001)001

On the other hand, consider a 0-brane sitting on top of a 2-brane in the closed string background described in the previous subsection. We wish to determine E_{bind}, the difference between the energy of this configuration and one in which the 0-brane is completely dissolved in the 2-brane. For this purpose we work in commutative variables. It is convenient to gauge away the bulk $B_{\mu\nu}$ field inducing an equal $F_{\mu\nu}$ field on the brane. Let the constant value of F be equal to B after the 0-brane has dissolved into the 2-brane. The energy of this dissolved state is

$$E = \frac{1}{g_{str}(2\pi)^2\alpha'^{\frac{3}{2}}} \int d^2x \sqrt{\det\left(g + 2\pi\alpha' B\right)} = \frac{1}{(2\pi)^2 g_{str}\alpha'^{3/2}} \int d^2x \sqrt{g^2 + (2\pi\alpha' B)^2}\,. \tag{4.5}$$

In the limit of large noncommutativity (4.2), (4.5) may be expanded as

$$E = \frac{1}{g_{str}\sqrt{\alpha'}} \frac{1}{2\pi} \int d^2x B \left(1 + \frac{1}{2}\frac{g^2}{(2\pi\alpha' B)^2} + \cdots\right). \tag{4.6}$$

Removing a unit of D0-brane charge from the constant value of the background F field on the brane, $((1/2\pi)\int d^2x \Delta F = -1)$ lowers the energy of the 2-brane by

$$\Delta E = \frac{1}{g_{str}\sqrt{\alpha'}} \left(1 - \frac{1}{2}\frac{g^2}{(2\pi\alpha' B)^2}\right). \tag{4.7}$$

Thus

$$E_{bind} = E(D0) - \Delta E = \frac{1}{g_{str}\sqrt{\alpha'}} - \Delta E = \frac{1}{2 g_{str}\sqrt{\alpha'}} \frac{g^2}{(2\pi\alpha' B)^2} \tag{4.8}$$

in precise agreement with the energy (4.4) of the solution (2.5).

4.3 Matching spectra

We have interpreted the solution (2.5) as m D0-branes sitting outside a D2-brane that carries large D0-brane charge density. In this subsection we will match the spectrum of fluctuations about (2.5) with the spectrum of the free 0-2 conformal field theory. Of course we expect complete agreement only in the scaling limit (4.2); away from this limit the full 0-2 CFT has a Hagedorn spectrum of string scale states. Rather surprisingly, however, the CFT also has a harmonic oscillator tower of states whose energies are parametrically small in string units.[8] These states have masses proportional to $1/\theta$ rather than $1/\alpha'$ and survive the scaling limit. They match perfectly with the spectrum of fluctuations about the solution (2.5) derived in subsection 2.3. In particular, the B field shifts the 0-2 tachyon mass-squared to just below zero, in such a way that it remains finite in the scaling limit.

[8]This was already observed in [13]. That paper also computes the spectrum of the 0-2 CFT described in this section. The computation of the 0-2 spectrum in appendix C was also carried out in [24]. Closely related computations in the 0-4 system are also described in [19], whose notations and conventions we follow.

JHEP04(2001)001

Let us first recapitulate from subsection 2.3 the relevant properties of the soliton configuration corresponding to m 0-branes sitting on top of the 2-brane, i.e. the solution (2.5) with $\varphi_a^i = 0$. Using (4.1), and in particular

$$\frac{1}{G\theta} = \frac{g}{4\pi^2\alpha'^2 B}, \tag{4.9}$$

the fluctuation spectrum derived in section 2.2 consists of

(a) massless $m \times m$ transverse scalars, on the 0-branes.

(b) massless fluctuations of the gauge field and transverse scalars on the 2-brane.

(c) a single complex tachyon (in the fundamental of U(m)) with mass $m^2 = -g/(4\pi^2\alpha'^2 B)$ and a harmonic tower of states with $m^2 = (2k+1)g/(4\pi^2\alpha'^2 B)$ $k = 1, 2, \ldots$ (from the gauge modes). One additional such tower with states with $m^2 = (2k+1)g/(4\pi^2\alpha'^2 B)\, k = 0, 1, \ldots$ for each transverse scalar.

We now consider the direct computation of the 0-2 spectrum from the free conformal field theory that describes this system. Our detailed analysis of the spectrum follows that given for the 0-4 system in [19]. Here we summarize this analysis; some additional details are presented in appendix C.

The spectrum of 0-0 strings is unaffected by the B field. Only the massless states are light in string units; their spectrum is exactly as in (a) above. Similarly the only light 2-2 states are the massless fields, and they match precisely with (b) above. Now consider 0-2 strings. The part of the conformal field theory that involves the DD and NN directions (we work in the covariant formalism) is unaffected by the magnetic field, and may be dealt with as usual. The interesting effects occur in the N-D directions (directions 1 and 2). The B field modifies boundary conditions of these strings, influencing their mode expansion. It turns out that, in the NS sector, the fields $X = x^1 + ix^2$, $\bar{X} = X^\dagger$, $\psi = \psi^1 + i\psi^2$ and $\bar{\psi}$ are mode expanded in terms of the operators $\alpha_{n+\nu}$, $\bar{\alpha}_{n-\nu}$, $\psi_{n+\nu-1/2}$ and $\bar{\psi}_{n-\nu+1/2}$ respectively, where n runs over all integers and, as usual, the subscript of an oscillator field represents its L_0 eigenvalue, $[L_0, \beta_r] = -r\beta_r$ (β represents any of the oscillators above). The constant ν depends on the magnetic field, and, in the limit (4.2), is given by

$$\nu = 1 - \frac{1}{\pi b}, \qquad b = \frac{2\pi\alpha' B}{g}. \tag{4.10}$$

Summing up all zero point energies, the energy of the vacuum state is [19],

$$E_{\text{vac}} = -\frac{1}{2}\left(\left|\nu - \frac{1}{2}\right| + \frac{1}{2}\right), \tag{4.11}$$

and, at large b, $E_{\rm vac} = -(1/2)(1-1/(\pi b))$. The state $|0\rangle$ is thus a spacetime tachyon with string scale mass. However, the GSO projection removes from the spectrum[9] $|0\rangle$ and all other states with even fermion number. The lowest energy state retained by the GSO projection is $\bar\psi_{-\nu+1/2}|0\rangle$ and has energy

$$E_{\rm vac} + \nu - \frac{1}{2} = -\frac{1}{2\pi b}\,. \tag{4.12}$$

This state corresponds to a spacetime tachyon. Note that its mass is parametrically small in string units, due to a cancellation of the leading terms. Another light state is $\psi_{-1+(\nu-1/2)}|0\rangle$, the energy of the state is $3/(2\pi b)$. Other states created by ND fermion oscillators have large, string scale masses. Turning to the remaining fields in the CFT, any of the lowest transverse (NN or DD) fermionic oscillators create states $\psi^\mu_{-1/2}|0\rangle$ with small positive mass, $1/(2\pi b)$. Finally, there exists a single bosonic oscillator $\alpha_{-1+\nu}$ whose energy $1/(\pi b)$ is parametrically small in string units. This oscillator turns each of each of $\bar\psi_{-\nu+1/2}|0\rangle$, $\psi_{-1+(\nu-1/2)}|0\rangle$, and $\psi^i_{-1/2}|0\rangle$ into the base of a harmonic oscillator tower of light states created by $\bar\psi_{-\nu+1/2}$, $\psi_{-1+(\nu-1/2)}$, and ψ^μ acting on $|k\rangle = 1/\sqrt{k!}\alpha^k_{-1}|0\rangle$.

Physical states $|\varphi\rangle$ are those

(a) that obey $L_{\{m>0\}}|\phi\rangle = 0 = G_{\{m>0\}}|\phi\rangle$, (where L_m, G_m are modes of the world-sheet energy momentum tensor and supersymmetry current, see appendix C),

(b) modulo states that are pure gauge, i.e. are of the form

$$|\phi\rangle = L_{\{m<0\}}|\chi\rangle \qquad \text{or} \qquad |\phi\rangle = G_{\{m<0\}}|\chi\rangle\,, \tag{4.13}$$

for some $|\chi\rangle$.

A general coherent state can be expanded as

$$|\phi\rangle = \sum_{k=0}^{\infty}\{W_k\psi_{-1+(\nu-1/2)} + T_k\bar\psi_{-\nu+1/2} + (\chi_\mu)_k\psi^\mu_{-1/2}\}|k\rangle\,, \tag{4.14}$$

where W_k, T_k and χ_μ are functions of time. The constraints (a) and (b) above are non-trivial for operators $G_{(\pm 1/2)}$. $G_{1/2}|\phi\rangle = 0$ implies that

$$\sqrt{k}W_{k-1} + \sqrt{k+1}T_{k+1} - \partial_t(\chi_t)_k = 0\,. \tag{4.15}$$

Acting by $G_{-1/2}$ on $\phi_k|k\rangle$, where $\phi_k = \phi_k(t)$, we conclude that

$$(W_k, T_k, \chi^0_k) \sim (W_k + \sqrt{k+1}\varphi_{k+1}, T_k + \sqrt{k}\varphi_{k-1}, \chi^0_k - \partial_0\varphi_k) \tag{4.16}$$

[9] The opposite GSO projection that retains states with even fermion number and projects out states with odd fermion number yields the conformal field theory for an anti zero brane sitting on top of the 2-brane [19].

where φ_k are arbitrary. We can use (4.16) to set $(\chi_t)_k = 0$, $(k = 0, 1, \ldots)$, and then the stringy constraint (4.15) implies

$$\sqrt{k+1}T_{k+1} + \sqrt{k}|W_{k-1} = 0\,, \qquad k = 0, 1, \ldots \tag{4.17}$$

eliminating roughly one of the two towers.[10] All $\chi^i_k (i = 1, \ldots)$ states are physical.

In summary, the spectrum of light 0-2 strings contains:

(i) A tachyon T_0 of squared mass $-1/(2\pi\alpha' b) = -g/(4\pi^2\alpha'^2 B)$,

(ii) Tower of massive states, with $m^2_k = (2k+1)/(2\pi\alpha' b)$ $k = 1, 2, \ldots$, which are linear combinations of 0-2 Fock-space states $\frac{1}{\sqrt{2k+1}}(\sqrt{k}T_{k+1} - \sqrt{k+1}W_{k-1})$.

(iii) A tower of states χ^i_k, for $k = 0, 1 \ldots$, of masses $m^2_k = (2k+1)/(2\pi\alpha' b)$.

The spectrum of light states in precise agreement with the spectrum of oscillations above the soliton.

Finally when the zero brane is displaced from the 2 brane by a physical distance x^1 (say in the 1st transverse direction), all squared masses computed above are then increased by $(x^1)^2/(4\pi^2\alpha'^2)$. Making the usual identification $x^i = 2\pi\alpha'\varphi^i$, this matches the shift in squared mass of fluctuation modes of the soliton (2.5) on giving φ^i an expectation value.

4.4 Interpretation in matrix theory

It is well known that a D2-brane with a background magnetic field can be constructed out of an infinite number of D0-branes [25]; a D2-brane in the 12 directions may be represented as a matrix configuration

$$X^1 = X^1_0\,, \qquad X^2 = X^2_0\,, \tag{4.18}$$

where X^1_0 and X^2_0 are infinite dimensional matrices obeying

$$[X^1_0, X^2_0] = i\theta\,. \tag{4.19}$$

Fluctuations about this soliton describe a noncommutative gauge theory [26]–[33], [34]. On the other hand matrices corresponding to m 0-branes at specific spatial postions are simply diagonal $m \times m$ blocks, whose eigenvalues give the postions of each of these 0-branes. Appending such an $m \times m$ block to the top left corner of the matrix representing a D2-brane (and setting all off diagonal elements to zero) yields a system consisting of m D0-branes plus a D2-brane. This configuration is clearly a solution to the equations of motion; it would be natural to expect it to represent the soliton in the noncommutative field theory that corresponds to m D0-branes sitting on the D2-brane.

[10] Indeed the states identified as pure gauge in this procedure are in precise correspondence with the off diagonal modes of the fluctuation gauge field that were set to zero by the Gauss Law constraint in the fluctuation analysis of subsection 2.3.

Indeed C_0 (see (2.17)) is precisely of this form. We regard this observation as providing further evidence, from a different viewpoint, for our interpretation of the soliton (2.5).[11]

4.5 The D-string

Eq. (2.5) may trivially be lifted to a $p-2$ dimensional soliton of $p+1$ dimensional noncommutative Yang Mills with spatial noncommutativity in at least two directions. The solution may then be interpreted as m D($p-2$)-branes sitting outside a Dp-brane.

The case $p=3$ is of particular interest. In this case (2.5) is a string like soliton in $3+1$ dimensional noncommutative Yang Mills, and may be interpreted as a D-string parallel to the world-volume of a D3-brane. This is to be contrasted with the Gross-Nekrasov string soliton [3, 4] which may be interpreted as a D-string piercing (or ending on) the D3-brane at an angle. We note that the tension (2.12) of our non-BPS solution (2.5) is exactly half of that of the BPS Gross-Nekrasov string.

$3+1$ dimensional Yang Mills is known to be S-dual to $3+1$ dimensional NCOS theory [35]. Under this S-duality, the string soliton (2.5) maps into the NCOS string. It is easy to verify that the tension of the NCOS string at weak NCOS coupling ([35, eq. (2.11)]) matches its tension at strong NCOS coupling (eqn. (2.12) of this paper), suggesting non renormalization of the NCOS string tension.

4.6 The 0-4 system

In section 3.1 we conjectured that the solution (3.4) represents an anti 0-brane sitting on top of a 4-brane in the appropriate B field. In this subsection we will compare the binding energy of (3.4) with that of an anti 0-brane sitting on top of a 4-brane in a large B field. The two binding energies match, providing evidence for our conjecture.

For simplicity we restrict attention to a closed string background with block diagonal $B_{\mu\nu}$ and $g_{\mu\nu}$ moduli. Let the B field and metric along the directions of the brane be given by $g_{\mu\nu} = (g_1 I_2, g_2 I_2)$ and $B_{\mu\nu} = (B_1\epsilon, B_2\epsilon)$, respectively. In the limit of large noncommutativity

$$\alpha' B_i \gg g_i \tag{4.20}$$

the Yang Mills coupling g_{YM}^2, open string metric $G_{\mu\nu} = (G_1 I_2, G_2 I_2)$ and noncommutativity parameter $\Theta^{\mu\nu} = (\theta_1\epsilon, \theta_2\epsilon)$ on the 4-brane are [19]

$$\begin{aligned} \theta_i &= \frac{1}{B_i} \\ G_i &= \frac{(2\pi\alpha' B_i)^2}{g_i} \\ g_{YM}^2 &= g_{str}(2\pi)^4 \alpha'^{5/2} \frac{B_1 B_2}{g_1 g_2}\,. \end{aligned} \tag{4.21}$$

[11] We thank Per Kraus for discussions on this connection.

The binding energy (3.9) of the solution (3.4), written as a function of closed string moduli is

$$E_{\rm bind} = \frac{1}{8\pi^2\alpha'^{5/2}}\left[\frac{g_1}{B_1} - \frac{g_2}{B_2}\right]^2 . \tag{4.22}$$

On the other hand, the binding energy of the D0-D4 system is the energy of a 0-brane plus the energy of a 4-brane minus the energy of the 0-4 bound state, all in the appropriate background B field. In the limit (4.2) it is also given by (4.22), see [36].[12]

5. Tachyon condensation

The condensation of the open string tachyon on a D-brane in bosonic string theory and on a D-$\bar{\rm D}$ system in type II theory has received much recent attention (see, for instance, [37] and references therein). One of the difficulties encountered in these discussions is the lack of a parameter controlling the approximation leading to the tachyon lagrangian. The 0-2 system we have studied in this paper also has a world-volume tachyon, and can be regarded as a toy laboratory [13] for the more difficult and interesting D-$\bar{\rm D}$ system. In the 0-2 context there is a small parameter, namely the ratio of the string scale to the noncommutativity scale, which can be used to control the approximations. In the scaling limit in which this ratio goes to zero, the theory contains a tower of positive mass states as well as the tachyon. Using the construction of the 0-brane as a soliton (2.5) in noncommutative Yang Mills, the lagrangian can be exactly constructed and tachyon condensation reliably studied.

5.1 General considerations

We have argued that m coincident D0-brane on a 2-brane may be represented as the soliton (2.16) (with $\varphi_0^i = 0$). The gauge field C for this solution is

$$C_0 = \begin{pmatrix} 0 & 0 \\ 0 & a^\dagger \end{pmatrix} . \tag{5.1}$$

We parameterize the arbitrary C field by the fluctuation fields of subsection 2.3

$$C = \begin{pmatrix} A & W \\ \bar{T} & a^\dagger + D \end{pmatrix} . \tag{5.2}$$

The fields A, D and W, T are identified with 0-0, 2-2, and 0-2 strings in the conformal field theory description of the 0-2 system.

The soliton (2.5) is unstable to the spreading out of flux; at the endpoint of tachyon condensation the soliton flux is spread evenly over the infinite 2-brane and is

[12] See eq. (4.10) of that paper. The translation of notation is as follows. $2\pi^2 g^2$ in [36] should be identified g_{str} in this paper , while b_i in [36] is to be identified with $2\pi\alpha' B_i$ in this paper.

[13] See [38]–[40] for other models which capture some aspects of tachyon condensation.

consequently invisible. The final configuration is $C = a^\dagger$.[14] About this configuration it is most natural to parameterize $C = a^\dagger + A'$ where A' is a new field. The endpoint of tachyon condensation is described by the CFT of a single 2-brane, A' is identified with the 2-2 gauge boson of this CFT.

Note that the 2-2 string modes in the CFT after tachyon condensation include all the 0-0, 0-2, 2-0, and 2-2 strings of the CFT prior to tachyon condensation. Thus, in the process of tachyon condensation, 0-0 and 0-2 modes are absorbed into the 2-2 continuum. In this respect tachyon condensation in the 0-2 system appears qualitatively different from tachyon condensation in a p-$\bar{p}$ system. In the latter case there appears to be no continuum for the p-$\bar{p}$ modes to disappear into. Restated, the decay of the 0-brane into 'nothing' in the 0-2 system is not mysterious once the 0-brane is constructed as a soliton on the 2-brane.

5.2 The tachyon potential

We now study the scalar potential in this 0-2 system.[15] For notational convenience we set θ to unity through the bulk of this sub-section.

The initial state $C = (S^\dagger)^m a^\dagger S^m$ decays to the final state $C = a^\dagger$ on exciting the tachyonic mode $T = C_{m,m-1}$. Note that the tachyonic mode and the nonzero matrix elements in the initial and final state are all of the form $C_{i+1,i}$. One might thus suspect that it is possible to set all C matrix elements not of this form to zero through the entire process of tachyon condensation. This is indeed the case,[16] as (2.2) admits a consistent truncation to these modes.

For clarity we specialize to $m = 1$. The potential expanded about the solitonic solution (2.5) may be written as a function of the tachyon T and the 2-2 gauge fields $D_{i,i-1}$ (using the notation of appendix A).

$$V = \frac{\pi}{g_{YM}^2}\Bigg[\left[|T|^2 - 1\right]^2 + \left[|T|^2 - |D_{1,0} + 1|^2 + 1\right]^2 + \\ + \sum_{i=1}\left[|D_{i,i-1} + \sqrt{i}|^2 - |D_{i+1,i} + \sqrt{i+1}|^2 + 1\right]^2\Bigg]. \tag{5.3}$$

[14] Note that 0-0, 0-2 and 2-2 fields are all excited at the endpoint of tachyon condensation and at that point (see appendix A for notation)

$$A = P_{m-1} a^\dagger P_{m-1}\,, \quad D = S^m a^\dagger (S^\dagger)^m - a^\dagger \quad W = 0\,, \quad T = 0 \quad \text{except } T_{m-1,0} = \sqrt{m}\,.$$

[15] The tachyon potential in the 0-4 system in a B field has recently been studied in [36].

[16] In order to demonstrate this we assign the the fields C_{ij} and C^*_{km} 'angular momentum' quantum numbers $i - j$ and $m - k$ respectively. With this assignment the potential conserves angular momentum. All terms in the potential are the product of an equal number of C and a C^* fields. Angular momentum conservation prohibits linear coupling of 'other fields' to C (C^*) fields of angular momentum 1 (-1).

JHEP04(2001)001

An unstable extremum of (5.3) that corresponds to an undissolved 0-brane on the 2-brane is

$$T = D_{i,i-1} = 0\,. \tag{5.4}$$

It decays into the stable extremum

$$|T| = 1\,, \qquad D_{i,i-1} = \sqrt{i+1} - \sqrt{i}\,. \tag{5.5}$$

It is easy to see that (5.5) corresponds to $C = a^\dagger$, the 2-brane vacuum.

It is possible to integrate out the fields $D_{i,i-1}$ and obtain the potential V as a function of the tachyon alone. Minimizing V w.r.t $D_{i,i-1}$ we find

$$|D_{i,i-1} + \sqrt{i}|^2 = |T|^2 + i \tag{5.6}$$

which sets all except the first term (5.3) to zero. Restoring θ, the potential thus takes the simple form[17]

$$V = \frac{\pi\theta}{g_{YM}^2}\left[|T|^2 - \frac{1}{\theta}\right]^2\,. \tag{5.7}$$

To be precise, in obtaining (5.7) we have chosen the branch of the solution $D_{i,i+1}(T)$ that minimizes rather than extremizes (5.3). An infinite number of other branches exist (and lead to the m D0-brane solutions). (5.7) is the appropriate branch to examine the decay of the single D0-brane into the vacuum. On this branch the gauge field C corresponding to a static T[18] is

$$C = \sum_{n=0}^{\infty} |n+1\rangle\langle n| \sqrt{|T|^2 + n} \tag{5.8}$$

and

$$F = (1 - |T|^2)|0\rangle\langle 0|\,. \tag{5.9}$$

If we allow the tachyon to roll from $T = 0$ to any particular T and hold it at that value the localized component of the flux, it is given by (5.9) once the dust has settled. Thus as the tachyon rolls from $|T| = 0$ to the minimum at $|T| = 1$, it radiates flux away to infinity. At $|T| = 1$, the endpoint of tachyon condensation all the flux has been radiated away and the D0-brane has dissolved completely into the D2-brane.

In the case of the D2-D4 system, there is a $2+1$ dimensional complex tachyon on the world volume of the D2-brane. It is tempting to speculate that there are finite energy solutions with one unit of magnetic flux and winding of the tachyon which correspond to a D0-D4 system.

[17] A quartic tachyon potential was also obtained in [13], (see equation (2.8)) using scattering calculations in string theory, strengthening our identification of the fluctuation modes of subsection 2.3 with 0-2 strings.

[18] T is held static by an external source.

Acknowledgments

We are grateful to J. David, J. Harvey, F. Larsen, E. Martinec, A. Rajaraman, M. Rangamani, A. Sen, J. Troost and especially P. Kraus and J. Maldacena for useful discussions. This work was supported in part by DOE grant DE-FG02-91ER40654, NSF grants DMS 9709694, PHY 9802709 and the Harvard Society of Fellows.

A. The small fluctuation analysis

We examine quadratic fluctuations about the charge m solution (2.5). For simplicity we will first consider the special case $\varphi_0^i = 0$, $c^a = 0$.

A.1 $\varphi^i = 0$

For convenience we set $\theta = 1$ in the intermediate stages of the computation. θ is restored in the final answer by dimensional analysis.

Let

$$\begin{aligned} C &= C_0 + \delta C \\ \varphi^i &= \delta\varphi^i \,. \end{aligned} \tag{A.1}$$

As $P_{m-1}C_0 = 0 = C_0 P_{m-1}$, it is convenient to decompose the fluctuations as

$$\begin{aligned} \delta C &= A + W + \bar{T} + (S^\dagger)^m D S^m \\ \delta\varphi^i &= \chi^i + \psi^i + \bar{\psi}^i + (S^\dagger)^m \gamma^i S^m \end{aligned} \tag{A.2}$$

where

$$\begin{aligned} A &= P_{m-1}\delta C P_{m-1}\,, & W &= P_{m-1}\delta C(1-P_{m-1})\,, \\ \bar{T} &= (1-P_{m-1})\delta C P_{m-1}\,, & (S^\dagger)^m D S^m &= (1-P_{m-1})\delta C(1-P_{m-1}) \end{aligned} \tag{A.3}$$

and

$$\begin{aligned} \chi^i &= P_{m-1}\delta\varphi^i P_{m-1}\,, \qquad \psi^i = P_{m-1}\delta\varphi^i(1-P_{m-1})\,, \\ (S^\dagger)^m\gamma^i S^m &= (1-P_{m-1})\delta\varphi^i(1-P_{m-1})\,. \end{aligned} \tag{A.4}$$

To quadratic order in fluctuations, the potential energy in (2.13) is a linear combination of $(1/2)\left([C,\bar{C}] + (1/\theta)\right)^2$ and $\mathrm{Tr}[C,\varphi^i][\varphi^i,\bar{C}]$, which may be expanded as

$$\begin{aligned} \frac{1}{2}\,\mathrm{Tr}\left([C,\bar{C}] + \frac{1}{\theta}\right)^2 &= \frac{m}{2\theta^2} + \mathrm{Tr}(W\bar{C}_0 - TC_0)(C_0\bar{W} - \bar{C}_0\bar{T}) + \mathrm{Tr}(W\bar{W} - \bar{T}T) + \\ &\quad + \frac{1}{2}\,\mathrm{Tr}([a^\dagger,\bar{D}] + [D,a])^2\,, \end{aligned} \tag{A.5}$$

$$\mathrm{Tr}[C_0,\varphi^i][\varphi^i,\bar{C}_0] = \mathrm{Tr}\left((C_0\bar{C}_0 + \bar{C}_0 C_0)\bar{\psi}^i\psi^i + [a^\dagger,\gamma^i][\gamma^i,a]\right)\,. \tag{A.6}$$

The fields A and χ^i do not appear in (A.5) and (A.6), and so are massless. The quadratic potential for the modes D and γ^i is precisely that for fluctuations of the vector and the transverse scalars of (2.2) about its vacuum. Now turn to the fluctuations in ψ^i. Expanding ψ^i

$$\psi^i = \sum_{k=0}^{\infty} \sum_{a=0}^{m-1} \psi^i_{ak} |a\rangle\langle k+m| \,, \tag{A.7}$$

the relevant term in (A.6) may be rewritten as

$$\sum_{k=0}^{\infty} (2k+1)|\psi^i_{ak}|^2 \,. \tag{A.8}$$

From (2.13) and restoring units, we conclude that the off diagonal fluctuations of each scalar field lead to a harmonic oscillator tower of states of mass

$$m_k^2 = (2k+1)\frac{1}{\theta} \,, \qquad k = 0, 1, \dots \tag{A.9}$$

We now determine the spectrum for the off diagonal gauge field modes W and T. Let

$$W_{ak} = \langle a|W|k+m\rangle \,, \qquad T_{ak} = \langle a|T|k+m\rangle \,, \tag{A.10}$$

where $a = 0, \dots, m-1$ and $k = 0, \dots, \infty$. The relevant terms in (A.5) take the form (suppressing the summation over a)

$$\sum_{k=0}^{\infty} (|\sqrt{k} W_{a,k-1} - \sqrt{k+1} T_{a,k+1}|^2) + \sum_{k=0}^{\infty} (|W_{a,k}|^2 - |T_{a,k}|^2) \tag{A.11}$$

which may be regrouped as

$$-|T_{a,0}|^2 + \sum_{k=1}^{\infty} (2k+1)|Y_{a,k}|^2 \,, \tag{A.12}$$

where we have defined the normalized fields

$$Y_{a,k} = \frac{1}{\sqrt{2k+1}} (\sqrt{k+1} W_{a,k-1} - \sqrt{k} T_{a,k+1}) \,. \tag{A.13}$$

Restoring units, and using (2.13), we conclude that the spectrum of off diagonal gauge field fluctuations consists of a single complex tachyon of mass $-1/\theta$, a harmonic oscillator tower of complex modes with masses

$$m_k^2 = (2k+1)\frac{1}{\theta} \,, \qquad k = 1, 2, \dots \tag{A.14}$$

and an infinite number of zero modes corresponding to the orthogonal linear combinations $Z_{a,k}$

$$Z_{a,k} = \frac{1}{\sqrt{2k+1}} (\sqrt{k} W_{a,k-1} + \sqrt{k+1} T_{a,k+1}) \qquad k = 0, 1, \dots . \tag{A.15}$$

In fact, the $Z_{a,k}$ fields are not dynamical as a consequence of the Gauss law constraint (2.15). Gauss' law, (2.15) implies that to first order in fluctuations δC and $\delta\varphi$ about a time independent configuration C, φ

$$\partial_t \left([\bar{C}, \delta C] + [C, \delta\bar{C}] + \sum_{i=1}^{t} [\varphi^i, \delta\varphi^i] \right) = 0\,. \tag{A.16}$$

Substituting $\delta C = W + \bar{T}$ into (A.16) and using (A.10) yields

$$\partial_t \left[Z_{ak}|a\rangle\langle k+m| - Z^*_{ak}|k+m\rangle\langle a| \right] = 0\,, \tag{A.17}$$

implying that Z_{ak} are constant in time. Thus Z_{ak} are integration constants and not degrees of freedom and can be gauged to zero[19].

A.2 $\varphi^1 \neq 0$

In the rest of this appendix we will describe how the spectrum of fluctuations is altered upon turning on a vev for φ_1 in the solution (2.16), for simplicity leaving $c^a = 0$, $\varphi^i = 0$ $(i = 2, \ldots, t)$. Let

$$\varphi^1 = \sum_{a=0}^{m-1} \varphi_a |a\rangle\langle a| \tag{A.20}$$

where φ_a are arbitrary nonzero real numbers. The analysis proceeds as above, with the following modifications.

First consider fluctuations of the fields A_{ab} and χ_{ab}. The Gauss Law constraint (A.16) implies

$$\partial_t \left([\chi^1, \varphi^1] \right) = 0 \tag{A.21}$$

which (assuming $\varphi_a \neq \varphi_b$ for $a \neq b$) implies that physical fluctuations of χ^1 are diagonal. On the other hand the remaining fields A_{ab} and χ^i_{ab} $(i = 2, \ldots, t)$ are now

[19] In order to make this completely manifest we expand the arbitrary off diagonal gauge fluctuation in terms of $T_{a,0}$, $Y_{a,k}$ and $Z_{a,k}$

$$T^*_{a,o}|m\rangle\langle a| + \sum_{a=0}^{m-1} \left[\sum_{k=1}^{\infty} \frac{1}{\sqrt{2k+1}} \left(\sqrt{k+1}\, Y_{ak}|a\rangle\langle k+m-1| - \sqrt{k}\, Y^*_{ak}|k+m+1\rangle\langle a| \right) + \right. \tag{A.18}$$
$$\left. + \sum_{k=0}^{\infty} \frac{1}{\sqrt{2k+1}} \left(\sqrt{k}\, Z_{ak}|a\rangle\langle k+m-1| + \sqrt{k+1}\, Z^*_{ak}|k+m+1\rangle\langle a| \right) \right],$$

and note that all terms involving Z_{ak} in (A.18) are of the form $[\lambda, C_0]$ where λ is the anti-hermitian operator

$$\lambda = \frac{1}{\sqrt{2k+1}} \sum_{k=0}^{\infty} \sum_{a=0}^{m-1} \left(Z_{ak}|a\rangle\langle k+m| - Z^*_{ak}|k+m\rangle\langle a| \right). \tag{A.19}$$

Thus Z_{ak} can be set to zero at $t = 0$ by a gauge transformation, and the Gauss Law constraint ensures that Z_{ak} stays zero at all times.

massive, with squared mass (from the fourth and fifth terms in (2.13) respectively)

$$m^2(A_{ab}) = m^2(\chi^i_{ab}) = (\varphi_a - \varphi_b)^2\,. \tag{A.22}$$

The fields D and γ^i are unaffected by the vev for φ^1. We now turn to the off diagonal fields T, W, and ψ^i. The potential energy for off diagonal fluctuations receives several additions from the φ^1 vev. From the fifth term in (2.13) we find the additional contribution to the potential

$$\sum_{i=2}^{t}\sum_{a=0}^{m-1}(\varphi_a)^2\sum_{k=0}^{\infty}|\psi^i_{ak}|^2\,. \tag{A.23}$$

Thus the off diagonal fluctuations of the scalars φ^i $(i = 2\ldots t)$ pick up an additional mass

$$\Delta m^2(\psi^i_{ak}) = \varphi_a^2\,. \tag{A.24}$$

We now consider the gauge fields and ψ^1. In addition to the potential (computed above) at $\varphi_a = 0$

$$V = -|T_{a,0}|^2 + \sum_{k=1}^{\infty}(2k+1)|Y_{a,k}|^2 + \sum_{k=0}^{\infty}(2k+1)|\psi^1_{ak}|^2\,, \tag{A.25}$$

at nonzero φ_a we find the additional contributions (from the fourth term in (2.13))

$$\begin{aligned}\sum_{a=0}^{m-1}\sum_{k=0}^{\infty}&\left(\sqrt{2k+1}\varphi_a\left(\psi^1_{ak}Z^*_{ak} + (\psi^1_{ak})^*Z_{ak}\right) + \varphi_a^2\left(|W_{ak}|^2 + |T_{ak}|^2\right)\right) = \\ &= \sum_{a=0}^{m-1}\Bigg[\varphi_a^2|T_{a0}|^2 + \sum_{k=0}^{\infty}\left(\sqrt{2k+1}\varphi_a\left(\psi^1_{ak}Z^*_{ak} + (\psi^1_{ak})^*Z_{ak}\right)\right) + \\ &\qquad + \varphi_a^2\left(\sum_{k=1}^{\infty}|Y_{ak}|^2 + \sum_{k=0}^{\infty}|Z_{ak}|^2\right)\Bigg].\end{aligned} \tag{A.26}$$

The sum of the potentials in (A.25) and (A.26) may be rewritten as

$$\begin{aligned}V = \sum_{a=0}^{m-1}\Bigg[&(-1+\varphi_a^2)|T_{a,0}|^2 + \sum_{k=1}^{\infty}\left(2k+1+(\varphi_a)^2\right)|Y_{ak}|^2 + \\ &+ \sum_{k=0}^{\infty}(2k+1+(\varphi_a)^2)|V_{ak}|^2\Bigg],\end{aligned} \tag{A.27}$$

where

$$V_{ak} = \frac{1}{\sqrt{2k+1+(\varphi_a)^2}}\left(\sqrt{2k+1}\psi^1_{ak} + \varphi_a Z_{ak}\right). \tag{A.28}$$

Thus the spectrum of 0-2 states at $\varphi_a \neq 0$ is exactly that at $\varphi_a = 0$, shifted by $(\varphi_a)^2$. The existence of zero modes

$$U_{ak} = \frac{1}{\sqrt{2k+1+(\varphi_a)^2}} \left(\sqrt{2k+1} Z_{ak} - \varphi_a \psi^1_{ak} \right) \tag{A.29}$$

in the fluctuation spectrum is a consequence of the Gauss Law constraint (A.16) which, in the 0-2 sector yields

$$\partial_t \left[U_{ak} |a\rangle\langle k+m| + U^*_{ak} |k+m\rangle\langle a| \right] = 0 \,. \tag{A.30}$$

Thus U_{ak} is unphysical and can be set to zero by a choice of gauge.

B. The matrix T

In this appendix we construct a matrix T obeying

$$TT^\dagger = 1 \,, \qquad T^\dagger T = 1 - P_0 \,, \qquad P_0 T^\dagger = TP_0 = 0 \,, \tag{B.1}$$

where P_0 is the projector onto the lowest radial wave function of the two-dimensional harmonic oscillator. It is convenient to start with an angular momentum basis

$$|j, m\rangle = \frac{(a_1^\dagger)^{j+m} (a_2^\dagger)^{j-m}}{\sqrt{(j+m)!(j-m)!}} |0,0\rangle \,. \tag{B.2}$$

An integer ordering for this basis can be defined for example by

$$|j^2 + j + m\rangle \equiv |j, m\rangle \,. \tag{B.3}$$

In this basis the solution for T is

$$\langle k|T|l\rangle = \delta_{k,l+1} \,. \tag{B.4}$$

This solution is of course highly non-unique, but since different choices of T obeying (B.1) lead to the same field strength, one expects that they are related by $\mathrm{U}(\infty)$ transformations, as indeed can be verified.

C. Details of the 0-2 spectrum

In this appendix we present some details omitted in the discussion of the 0-2 CFT in subsection 4.3. This computation was also carried out in [24].

Let the string world-sheet be a strip, parameterized by coordinates $0 \leq \sigma \leq \pi$ and $-\infty < \tau < \infty$. The $x^{1,2}$ fields on a string beginning at the 0-brane and ending at the 2-brane obey the usual boundary conditions $\partial_\tau x^a|_{\sigma=0} = 0$ on the 0-brane and

$$g_{ab} \partial_\sigma x^b + 2\pi\alpha' B_{ab} \partial_\tau x^b |_{\sigma=\pi} = 0 \tag{C.1}$$

on the 2-brane. These boundary conditions may be diagonalized by working with the complex linear combination $X = x^1 + ix^2$. (C.1) implies the shifted mode expansion

$$X = \frac{i}{\sqrt{2}} \sum_{n=-\infty}^{\infty} [e^{i(n+\nu)(\tau+\sigma)} - e^{i(n+\nu)(\tau-\sigma)}] \frac{\alpha_{n+\nu}}{n+\nu}$$
$$\bar{X} = \frac{i}{\sqrt{2}} \sum_{n=-\infty}^{\infty} [e^{i(n-\nu)(\tau+\sigma)} - e^{i(n-\nu)(\tau-\sigma)}] \frac{\bar{\alpha}_{n-\nu}}{(n-\nu)}, \tag{C.2}$$

where $(\alpha_{n+\nu})^\dagger = \bar{\alpha}_{-n-\nu}$,

$$e^{2\pi i\nu} = -\frac{1+ib}{1-ib}, \qquad \nu \in [0,1) \tag{C.3}$$

and $b = \frac{2\pi\alpha' B_{12}}{g}$. In the limit (4.2) $b \gg 1$, and (C.3) simplifies to

$$\nu = 1 - \frac{1}{\pi b}. \tag{C.4}$$

World-sheet superpartners of x^1, x^2 fields are similarly affected by turning of B field in those directions. The moding of fermions in the NS sector is in the usual way shifted by a half-integer relative to that of the bosons. Puting $\psi_\pm = (\psi^1 + i\psi^2)_\pm$, we have

$$\psi_- = \frac{1}{\sqrt{2}} \sum_{n=-\infty}^{\infty} e^{i(n+\nu-\frac{1}{2})(\tau-\sigma)} \psi_{n+\nu-\frac{1}{2}}$$
$$\bar{\psi}_- = \frac{1}{\sqrt{2}} \sum_{n=-\infty}^{\infty} e^{i(n-\nu+\frac{1}{2})(\tau-\sigma)} \bar{\psi}_{n-\nu+\frac{1}{2}}, \tag{C.5}$$

where $\psi^\dagger_{n+\nu-\frac{1}{2}} = \bar{\psi}_{-n-\nu+\frac{1}{2}}$. ψ_+ is not independent, but is determined in terms of ψ_- by the NS boundary condition.

Upon quantization, α's and ψ's become creation and annihilation operators, whose non-vanishing (anti)commutators are

$$[\alpha_{m+\nu}, \bar{\alpha}_{n-\nu}] = (m+\nu)\delta_{m+n}\,; \qquad \{\psi_{m+\nu-1/2}, \bar{\psi}_{n-\nu+1/2}\} = \delta_{m+n}\,. \tag{C.6}$$

For completeness, we present expressions for the operators L_m, G_m. Then

$$L_m = \left(\sum_{n=-\infty}^{\infty} \alpha_{m+n+\nu}\bar{\alpha}_{-n-\nu} + \left(\frac{m-1}{2} + n + \nu\right) \psi_{m+n+\nu-\frac{1}{2}} \bar{\psi}_{-n-(\nu-\frac{1}{2})} \right) +$$
$$+\frac{1}{2} \left(\sum_{n=-\infty}^{\infty} \alpha_{m+n} \cdot \alpha_{-n} + \left(\frac{m-1}{2} + n\right) \psi_{m+n-\frac{1}{2}} \cdot \psi_{-n+\frac{1}{2}} \right), \tag{C.7}$$

where the last term runs over DD and NN oscilators, so for example $\alpha_m \cdot \alpha_n = -(\alpha_t)_m(\alpha_t)_n + (\alpha_i)_m \alpha^i_n$, and

$$G_{m+\frac{1}{2}} = \sum_{n=-\infty}^{\infty} \left(\alpha_{m+n+\nu} \bar{\psi}_{-n-(\nu-\frac{1}{2})} + \psi_{m+n+\nu+\frac{1}{2}} \bar{\alpha}_{-n-\nu} + \alpha_{m+n}\psi_{-n+\frac{1}{2}} \right). \tag{C.8}$$

References

[1] N. Nekrasov and A. Schwarz, *Instantons on noncommutative* $\mathbb{R}^4$ *and* $(2,0)$ *superconformal six dimensional theory*, *Comm. Math. Phys.* **198** (1998) 689 [hep-th/9802068].

[2] R. Gopakumar, S. Minwalla and A. Strominger, *Noncommutative solitons*, *J. High Energy Phys.* **05** (2000) 020 [hep-th/0003160].

[3] D.J. Gross and N.A. Nekrasov, *Monopoles and strings in noncommutative gauge theory*, *J. High Energy Phys.* **07** (2000) 034 [hep-th/0005204].

[4] D.J. Gross and N.A. Nekrasov, *Dynamics of strings in noncommutative gauge theory*, *J. High Energy Phys.* **10** (2000) 021 [hep-th/0007204].

[5] E. Witten, *Noncommutative Tachyons and String Field Theory*, hep-th/0006071

[6] D.P. Jatkar, G. Mandal and S.R. Wadia, *Nielsen-Olesen vortices in noncommutative abelian Higgs model*, *J. High Energy Phys.* **09** (2000) 018 [hep-th/0007078].

[7] G. Mandal and S.-J. Rey, *A note on D-branes of odd codimensions from noncommutative tachyons*, *Phys. Lett.* **B 495** (2000) 193 [hep-th/0008214].

[8] C.-G. Zhou, *Noncommutative scalar solitons at finite* θ, hep-th/0007255.

[9] A.S. Gorsky, Y.M. Makeenko and K.G. Selivanov, *On noncommutative vacua and noncommutative solitons*, *Phys. Lett.* **B 492** (2000) 344 [hep-th/0007247].

[10] C. Sochichiu, *Noncommutative tachyonic solitons: interaction with gauge field*, *J. High Energy Phys.* **08** (2000) 026 [hep-th/0007217].

[11] D. Bak and K. Lee, *Elongation of moving noncommutative solitons*, *Phys. Lett.* **B 495** (2000) 231 [hep-th/0007107].

[12] R. Gopakumar, S. Minwalla and A. Strominger, *Symmetry restoration and tachyon condensation in open string theory*, hep-th/0007226.

[13] E. Gava, K.S. Narain and M.H. Sarmadi, *On the bound states of* p- *and* $(p+2)$*-branes*, *Nucl. Phys.* **B 504** (1997) 214 [hep-th/9704006].

[14] K. Dasgupta, S. Mukhi and G. Rajesh, *Noncommutative tachyons*, *J. High Energy Phys.* **06** (2000) 022 [hep-th/0005006].

[15] J.A. Harvey, P. Kraus, F. Larsen and E.J. Martinec, *D-branes and strings as noncommutative solitons*, *J. High Energy Phys.* **07** (2000) 042 [hep-th/0005031].

[16] A. Sen, *Non-BPS states and branes in string theory*, hep-th/9904207.

[17] M.R. Douglas, *Branes within branes*, hep-th/9512077.

[18] C. Vafa, *Instantons on D-branes*, *Nucl. Phys.* **B 463** (1996) 435 [hep-th/9512078].

[19] S. Minwalla, M.V. Raamsdonk and N. Seiberg, *Noncommutative perturbative dynamics*, *J. High Energy Phys.* **02** (2000) 020 [hep-th/9912072].

[20] D. Bak, *Exact multi-vortex solutions in noncommutative abelian- Higgs theory*, *Phys. Lett.* **B 495** (2000) 251 [hep-th/0008204].

[21] A.P. Polychronakos, *Flux tube solutions in noncommutative gauge theories*, *Phys. Lett.* **B 495** (2000) 407 [hep-th/0007043].

[22] A. Hashimoto and I.W. Taylor, *Fluctuation spectra of tilted and intersecting D-branes from the Born-Infeld action*, *Nucl. Phys.* **B 503** (1997) 193 [hep-th/9703217].

[23] A. Connes, M.R. Douglas and A. Schwarz, *Noncommutative geometry and matrix theory: compactification on tori*, *J. High Energy Phys.* **02** (1998) 003 [hep-th/9711162].

[24] B. Chen, H. Itoyama, T. Matsuo and K. Murakami, *p-p' system with B field, branes at angles and noncommutative geometry*, *Nucl. Phys.* **B 576** (2000) 177 [hep-th/9910263]; *Worldsheet and spacetime properties of p-p' system with B field and noncommutative geometry*, *Nucl. Phys.* **B 593** (2001) 505 [hep-th/0005283].

[25] T. Banks, W. Fischler, S.H. Shenker and L. Susskind, *M-theory as a matrix model: a conjecture*, *Phys. Rev.* **D 55** (1997) 5112 [hep-th/9610043].

[26] M. Li, *Strings from IIB matrices*, *Nucl. Phys.* **B 499** (1997) 149 [hep-th/9612222].

[27] H. Aoki et al., *Noncommutative Yang-Mills in IIB matrix model*, *Nucl. Phys.* **B 565** (2000) 176 [hep-th/9908141].

[28] N. Ishibashi, *A relation between commutative and noncommutative descriptions of D-branes*, hep-th/9909176.

[29] N. Ishibashi, S. Iso, H. Kawai and Y. Kitazawa, *Wilson loops in noncommutative Yang-Mills*, *Nucl. Phys.* **B 573** (2000) 573 [hep-th/9910004].

[30] I. Bars and D. Minic, *Non-commutative geometry on a discrete periodic lattice and gauge theory*, *Phys. Rev.* **D 62** (2000) 105018 [hep-th/9910091].

[31] J. Ambjorn, Y.M. Makeenko, J. Nishimura and R.J. Szabo, *Finite N matrix models of noncommutative gauge theory*, *J. High Energy Phys.* **11** (1999) 029 [hep-th/9911041]; *Nonperturbative dynamics of noncommutative gauge theory*, *Phys. Lett.* **B 480** (2000) 399 [hep-th/0002158];
J. Ambjorn, K.N. Anagnostopoulos, W. Bietenholz, T. Hotta and J. Nishimura, *large-N dynamics of dimensionally reduced 4D* SU(N) *super Yang-Mills theory*, *J. High Energy Phys.* **07** (2000) 013 [hep-th/0003208];
J. Ambjorn, Y.M. Makeenko, J. Nishimura and R.J. Szabo, *Lattice gauge fields and discrete noncommutative Yang-Mills theory*, *J. High Energy Phys.* **05** (2000) 023 [hep-th/0004147].

[32] L. Alvarez-Gaume and S.R. Wadia, *Gauge theory on a quantum phase space*, *Phys. Lett.* **B 501** (2001) 319 [hep-th/0006219].

[33] A.H. Fatollahi, *Gauge symmetry as symmetry of matrix coordinates*, *Eur. Phys. J.* **C 17** (2000) 535 [hep-th/0007023].

[34] N. Seiberg, *A note on background independence in noncommutative gauge theories, matrix model and tachyon condensation*, *J. High Energy Phys.* **09** (2000) 003 [hep-th/0008013].

[35] R. Gopakumar, J. Maldacena, S. Minwalla and A. Strominger, *S-duality and noncommutative gauge theory*, *J. High Energy Phys.* **06** (2000) 036 [hep-th/0005048].

[36] J.R. David, *Tachyon condensation in the D0/D4 system*, *J. High Energy Phys.* **10** (2000) 004 [hep-th/0007235].

[37] A. Sen, *Some issues in non-commutative tachyon condensation*, *J. High Energy Phys.* **11** (2000) 035 [hep-th/0009038].

[38] D. Ghoshal and A. Sen, *Tachyon condensation and brane descent relations in p-adic string theory*, *Nucl. Phys.* **B 584** (2000) 300 [hep-th/0003278].

[39] B. Zwiebach, *A solvable toy model for tachyon condensation in string field theory*, *J. High Energy Phys.* **09** (2000) 028 [hep-th/0008227].

[40] J.A. Minahan and B. Zwiebach, *Field theory models for tachyon and gauge field string dynamics*, *J. High Energy Phys.* **09** (2000) 029 [hep-th/0008231].